Edge domination number of Jump Graph : A comparative Study

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Abstract

In this paper we are trying to find that the domination number of any graph is obtain by using familiar method as vertex domination techniques and edge domination techniques but from the both who one is faster than other?. By taking a very special graph for further investigation is Jump graph. Here we first obtain the results on domination number of Jump graph by vertex domination technique and later edge domination technique and make a latest results as conclusion.

Keyword
Jump Graph, Line Graph, Vertex domination, Edge domination.

1. **Introduction**

What is Jump Graph?
The Jump graph is Graph in which all the vertices in Graph consider as edges of graph and those vertices are not adjacent in given graph by obtaining the jump graph, all not adjacent vertices are adjacent together and the Jump graph we denote as $\mathbb{Z}(G)$.
In other words we are saying that $\mathbb{Z}(G)$ is complement of line graph we denote as $\mathcal{L}(G)$ is the simple graph in which all the vertices in graph G becomes the edges in $\mathcal{L}(G)$ with $e_1e_2 \in E(\mathcal{L}(G))$ when e_1 *and* e_2 having common end point in G.

For example consider the following graph G.

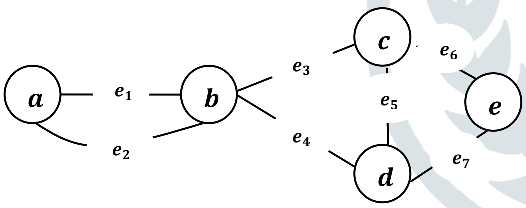

Figure-1: Graph G with five vertices

From the above graph G, the line graph $\mathcal{L}(G)$ as follows

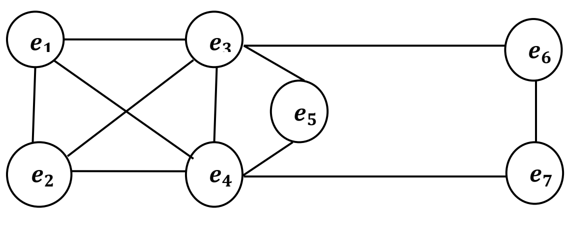

Figure-2: Line graph $\mathcal{L}(G)$

From the above graph $\mathcal{L}(G)$, the Jump graph $\mathbb{Z}(G)$ as follows,

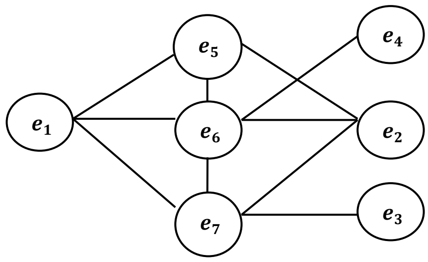

Figure-3: Jump graph $\mathbb{Z}(G)$ *from* $\mathcal{L}(G)$

It is clear that from the figure -1, the vertex domination number of graph G *is* $\gamma(G) = 1,$ similarly the vertex domination number of $\mathcal{L}(G)$ is $\gamma(\mathcal{L}(G)) = 2$ from figure-2 and the vertex domination number of $\mathbb{Z}(G)$ is $\gamma(\mathbb{Z}(G)) = 2$.

Now we consider another example of graph G with six vertices as follows:

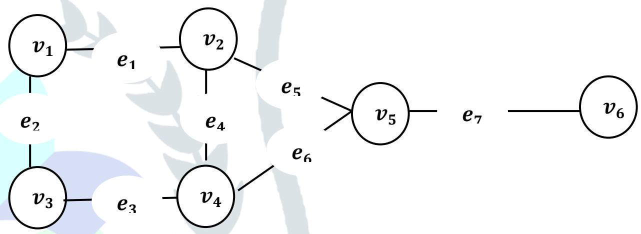

Figure-4: Graph G with six vertices

From the above graph G, the line graph $\mathcal{L}(G)$ as follows,

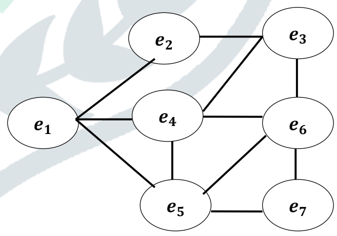

Figure-5: Line graph $\mathcal{L}(G)$

From the above graph $\mathcal{L}(G)$, the Jump graph $\mathbb{Z}(G)$ as follows,

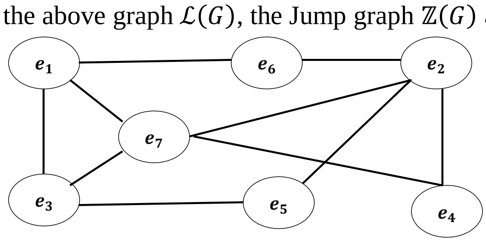

Figure-6: Jump graph $\mathbb{Z}(G)$ *from* $\mathcal{L}(G)$

As per the above discussion we see that the Jump graph and line graph both are derived from the given graph G are possible if and only if graph G containing more than 4 vertices otherwise one or more vertices are becomes pendant vertices. So here we are consider about the graph $G(V, E)$ with $|V| > 4$.

2. Some Basic Definitions and Known Results

How to define the domination number of Jump Graph according to vertex domination technique?
Let G be any graph and a set $D_v \subseteq V(G)$, set of vertices in G is said to be a dominating set, if every vertex not adjacent in V is adjacent in D_v and it is denoted by $\gamma(G)$ with minimum cardinality.
For the Jump graph $\mathbb{Z}(G)$, any non empty set D_v is said to be a dominating set of $\mathbb{Z}(G)$ if every vertex which is not adjacent in $V(\mathcal{L}(G))$ is adjacent in $\mathbb{Z}(G)$ and it is denoted by $\gamma(\mathbb{Z}(G))$, with minimum cardinality.

From the reference given in the book of O.Ore Theory of Graphs[1], mentioned the results as follows :

Theorem-1 [1] : Let G be a simple graph. If D_v is a minimal dominating set, then $V - D_v$ is a dominating set.

Theorem-2 [1] : If G be a simple graph then $\gamma(G) \le \frac{n}{2}$ where n indicates the number of vertices in G.

Theorem-3 [1] : If G be a simple graph then $2m \le n^2 - n$,where n indicates the number of vertices and m indicates number of edges in G.
From the reference given in the paper of Domination number of a Jump graph by Y.B.Maralabhavi, Anupama.S.B and Venkanagouda M. Goudar mentioned [2], the following results :

Theorem-4 [2]: For any connected graph $G(V,E)$ with $|V| = n \text{ and } |E| = m$ then $\gamma_v(G) + \gamma_v(\mathbb{Z}(G)) < \left(\frac{n+1}{2}\right)^2$. Where γ_v indicates the domination number of G with respect to vertex domination techniques.

3. Main Results:

According to above theorem, now we prove the same results by using edge domination techniques.
As per the definition of edge domination number as $D_e \subseteq E(G)$ is said to be a dominating set if every edge which is not incident in G is incident in D_e and it is denoted by $\gamma_e(G)$ with minimum cardinality. By the Hand shaking theorem of Graph Theory we know that $\sum d(v_i) = 2e$.

We claim that

Result-1: For any connected graph $G(V,E)$ with $|V| = n \text{ and } |E| = m$ then $\gamma_e(G) + \gamma_e(\mathbb{Z}(G)) < \left(\frac{m+1}{2}\right)^2$. Where γ_e indicates the domination number of G with respect to edge domination techniques.

Proof: Let G be any simple graph with $|V| = n$ and $|E| = m$ then $m \le \frac{n(n-1)}{2}$. And it is but obvious $\gamma_e(G) \le \min\{|D_e|, |E - D_e|$, follows that $\gamma_e(|\mathbb{Z}(G)|) \le \frac{m(m-1)}{4}$.
So

$$\therefore\ \gamma_e(G) + \gamma_e(|\mathbb{Z}(G)|) \le \frac{m}{2} + \frac{m(m-1)}{4}$$
$$\le \frac{m(m+1)}{4}$$
$$< \frac{(m+1)^2}{4}$$
$$< \left(\frac{m+1}{2}\right)^2.$$

Hence prove the result.

Result-2: For any connected graph $G(V,E)$ with $|V| = n \text{ and } |E| = m$ and diameter of graph G means $diag(G) \ge 2$, then $\gamma_e(|\mathbb{Z}(G)|) \ge 2$.

Proof: Let e_1e_2 be a path of maximum distance in graph G. then the distance of path is same as the diameter of graph G.
So we can distribute the above result into two cases:

Case-1: Suppose the path with maximum distance is 2.
Then in the Jump graph there is a set with maximum two edges are has to adjacent and generate a dominating set, therefore $\gamma_e(|\mathbb{Z}(G)|) = 2 - - - - - - (1)$

Case-2: Suppose path of length is greater than 2 more than 2 edges are adjacent together then it will create a dominating set with minimum cardinality is more than 2. Therefore $\gamma_e(|\mathbb{Z}(G)|) > 2 - - - - - - (2)$

Now from (1) and (2) we have $\gamma_e(|\mathbb{Z}(G)|) \ge 2$.

Result-3: For any connected graph $G(V,E)$ with $|V| = n \text{ and } |E| = m$ then $\gamma_e(|\mathbb{Z}(G)|) \ge 2$.

Proof : That is but obvious.

Result-4: If D_e be any dominating set of jump graph $\mathbb{Z}(G)$ such that $|D_e| = \gamma_e(\mathbb{Z}(G))$ then $E(\mathbb{Z}(G)) - D_e \le 2e_i, i = 1,2,\ldots m$. where G be any connected graph $G(V,E)$ with $|V| = n \text{ and } |E| = m$.

Proof: Since every edge in $E(\mathbb{Z}(G)) - D_e$ is adjacent to at least one edge in D_e, therefore we say that $E(\mathbb{Z}(G)) - D_e \le \sum d(v_i)$.
And hence according to Handshaking theorem of graph theory we have
$E(\mathbb{Z}(G)) - D_e \le \sum d(v_i) = 2e_i$.
$\Rightarrow E(\mathbb{Z}(G)) - D_e \le 2e_i$.

Result-5: For any tree T with diameter greater than 3 then $\gamma_e(|\mathbb{Z}(T)|) = 2$.

Proof: We verify earlier results if any tree with diameter more than 3 then definitely it will disconnected tree.

According to this result let e_1e_2 be any path with maximum length in a tree T. Let v_i be any pendent vertex and e_i be any pendent edge in tree T. Then the vertex set and edge set make a dominating set say D_e in $\mathbb{Z}(T)$.

Now all the vertices and edges are perform minimum dominating set. Hence $\gamma_e(|\mathbb{Z}(T)|) = 2$.

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