Vol. 8, Issue: 4, April: 2020
ISSN:(P) 2347-5404 ISSN:(O)2320 771X

# Domination Model of Graphical Method for Solving LPP

Tushar J Bhatt[1], Dr.G.C.Bhimani[2]

1. Assistant Professor in Mathematics, Atmiya University, Rajkot

2. Professor and Head, Department of Statistics, Saurashtra University, Rajkot.

**Abstract:**
*In this paper we can try to find a relationship between two different disciplines as theory of domination and operation research. In that concern we can try to make, a domination model for linear programming problem (LPP) solution method namely, Graphical method. By help of this domination model we can get basic feasible solution of LPP and hence we are trying to minimize the limitations of graphical method for solving LPP.***https://www.ijert.org/our-indexing***

**Keywords:** *Dominating set, Minimal dominating set, Minimum dominating set, Domination number, Linear programming problem (LPP), Mathematical modelling , Greatest integer function and congruence relation.*

## 1. Introduction

Here first we define the necessary terms those are mentioned as keywords, also give an example of each if necessary.

### *1.1 Definition*

**1.Dominating Set**

Consider G be a graph and V(G) be the set of all vertices of G. Let S be a sub-set of V(G). Then S is called a dominating set in the graph G if and only if for any $w \in V(G) - S$, we can find at least one vertex $v \in S$ such that w is attached to v.

For example, consider the sided graph **G**, according to the definition of dominating -set, the set $\mathbf{S} = \{\mathbf{v_1}, \mathbf{v_3}\}$ is a dominating set.

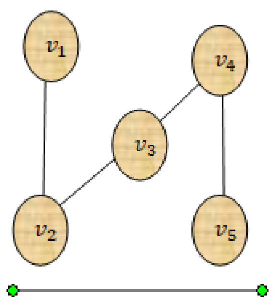



**Figure 1: Dominating set**

**2. Minimal Dominating Set**

Assume that G be a graph. A dominating set S in the graph G is called minimal dominating set in the graph G if and only if for any $v \in S$, $S - v$ is not a dominating set in the graph G. For example, consider the sided graph G, in which according to the definition of minimal dominating set, the set $S = \{v_1, v_4\}$ is a minimal dominating set.

Vol. 8, Issue: 4, April: 2020
ISSN:(P) 2347-5404 ISSN:(O)2320 771X

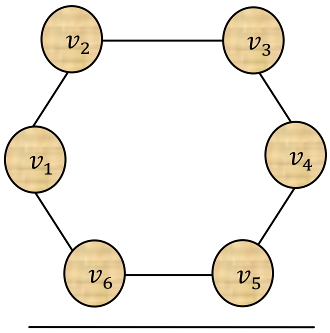



**Figure 2: Minimal Dominating set**

**3. Minimum Dominating Set**

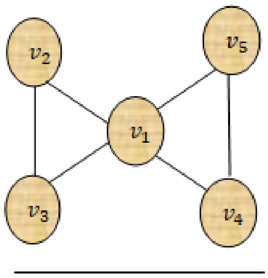



**Figure 3: Minimum Dominating set**

Consider the graph $G$. A dominating set S in G with minimum cardinality is called a minimum dominating set in the graph G.A minimum dominating set in G is also called a $\gamma$ $-$ set in the graph $G$. For example, consider the sided graph $G$, in which according to the definition of minimum dominating set, the set $S = \{v_1\}$ is a minimum dominating set.

**4. Domination Number**

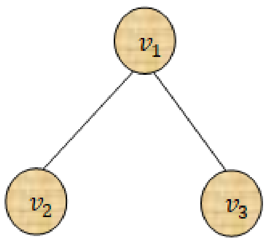



**Figure 4: Domination Number**

Consider $G$ be any graph and $S$ be a min. dominating set in $G$. Then $|S|$, cardinality of set $S$, is called the domination number of the graph and it is denoted by $\gamma$ (G).
**Remark. (a)** If S is a dominating set in graph G, then $\gamma(G) \leq |S|$.
**Remark. (b)** Every minimum dominating set in G is a minimal dominating set in G.
**Definition 1.5 (LPP):** The Linear Programming Problem is a special type of optimization problems . A general linear programming problem consists of maximizing or minimizing an objective function, subject to certain given constraints. It is concern with finding optimal value of an objective function of several variables subject to the conditions that the variables are non – negative and satisfying linear constraints.

Vol. 8, Issue: 4, April: 2020
ISSN:(P) 2347-5404 ISSN:(O)2320 771X

**For example**

**(a) Manufacturing Problems**

Determine the number of units that should be produced and sold in order to maximize profit, when each product requires a fixed manpower, machine hours and row materials.

**(b) Diet Problems**

Determine the amount of different kinds of to be included in the diet, minimizing the cost and subject to the availability of food and their prices.

**(c)Transportation Problems**

Determine a transportation schedule to find the cheapest way of transporting a product from plants or factories situated at different location to different markets.

## 5. Mathematical Models

Mathematical models are perhaps the most abstract of the four classifications. These models do not look like their real-life counterparts at all. Mathematical models are built using numbers and symbols that can be transformed into functions, equations, and formulas.

They also can be used to build much more complex models such as matrices or linear programming models. The user can then solve the mathematical model (seek an optimal solution) by utilizing simple techniques such as multiplication and addition or more complex techniques such as matrix algebra or Gaussian elimination.

Since mathematical models frequently are easy to manipulate, they are appropriate for use with calculators and computer programs. Mathematical models can be classified according to use (description or optimization), degree of randomness (deterministic and stochastic), and degree of specificity (specific or general). Following is a more detailed discussion of different types of mathematical models.

## 6. Types of Mathematical Models

There are two types of Mathematical models, defined as follows:

### *6.1 Descriptive Models*

Descriptive models are used merely to describe something mathematically. Common statistical models in this category include the mean, median, mode, range, and standard deviation. Consequently, these phrases are called "descriptive statistics." Balance sheets, income statements, and financial ratios are also descriptive in nature.

### *6.2 Optimization Models*

Optimization models are used to find an optimal solution. The linear programming models are mathematical representations of constrained optimization problems. These models share certain common characteristics. Knowledge of these characteristics enables us to recognize problems that can be solved using linear programming. For example, suppose that a firm that assembles computers and computer equipment is about to start production of two new types of computers. Each type will require assembly time, inspection time, and storage space. The amounts of each of these resources that can be devoted to the production of the computers is limited. The manager of the firm would like to determine the quantity of each computer to produce in order to maximize the profit generated by their sale.

**Note:** Here we are discussing about "Optimization Models" especially for Graphical method.

## 7. Greatest integer function

The Greatest Integer Function [X] indicates an integer part of the real number X which is nearest and smaller integer to X. It is also known as floor of X.

$\text{i.e.}: [X] = \text{The largest integer that is less than or equal to X.}$

| | | | | | |
|---|---|---|---|---|---|
| **X** | 3.3 | -8.0725 | 2 | 2.7 | -1.3 |
| [**X**] | 3 | -9 | 2 | 2 | -2 |

**For example**

**8. Congruence Relation**

The congruence relation is congruence modulo n on the set of integers. For a given integers $x$ and $y$ are called congruent modulo n means $x \equiv y \ (\text{mod } n)$.i. e.: $x - y$ Is divisible by n or $x$ and $y$ have the same remainder when divided by n.

**For example:** $37 \equiv 57 \ (\text{mod } 10)$ as $37 - 57 = -20$ is a multiple of 10.

## 2. Some Important Concepts

### *2.1 Stages of modeling*

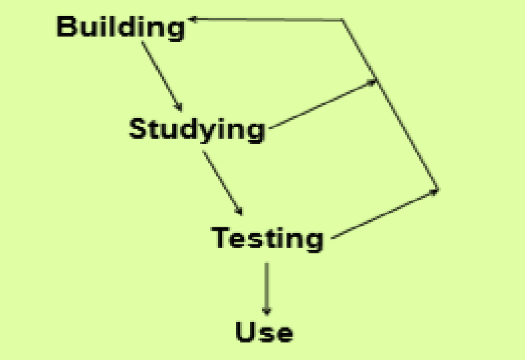



**Figure 5 : Stages of Mathematical Modeling**

It is helpful to divide up the process of modeling into four broad categories of activity, namely building, studying, testing and use. Although it might be nice to think that modeling projects progress smoothly from building through to use, this is hardly ever the case.

In general, defects found at the studying and testing stages are corrected by returning to the building stage. Note that if any changes are made to the model, then the studying and testing stages must be repeated.

A sided figure shows the pictorial representation of potential routes through the stages of modeling.

This process of repeated iteration is typical of modeling projects, and is one of the most useful aspects of modeling in terms of improving our understanding about how the system works.

### *2.2 Properties of LP Models*

**Proportionality**: The rate of change (Slope) of the objective function and the constraint equations or inequations with respect to the particular decision variable is constant.

**Additivity**: Terms in the objective function and constraint equations /inequations must be additive.

**Divisibility:** Decision variables can take on any fractional value and are therefore continuous as opposed to integer in nature.

**Certainty:** Value of the all model parameters are assumed to be known with certainty (non-probabilistic).

### *2.3 Formulation of LPP*

**I. Production Allocation Problem**

A manufacturer produce two types of models M and N each M model requires 4 hours grinding and 2 hours for polishing whereas each N model requires 2 hours of grinding and 5 hours for polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hrs a week and each polisher's works for 60 hrs a week. Profit on an M model is 3 RS. And on an N model is 4 Rs. whatever is produced in a week sold in the market. How should the manufacturers allocate this production capacity to the two types of models? So that he may make the maximum profit in a week?

**Solution**

Here the manufacturer's allocation problem contains

Vol. 8, Issue: 4, April: 2020
ISSN:(P) 2347-5404 ISSN:(O)2320 771X

**Decision Variables:**
$x_1$ = number of units produces by model M
$\mathbf{x_2}$ = number of units produces by model N

**Objective Function**
Maximize Profit
$\text{MaxZ} = 3x_1 + 4x_2$

**Constraints Identification**
Subject to constraints
$4x_1 + 2x_2 \leq 80$
$2x_1 + 5x_2 \leq 180$
and
$x_1, x_2 \geq 0.$

## 3. Main results

### *3.1 Domination technique to obtain optimum basic feasible solution for LPP by Graphical method*

- Techniques for Create the Domination Model
- Building

**Step-1:** To obtain a feasible region from Graphical method
**Step-2:** To made the graph from feasible region , all the coordinates consider as vertices for graph G.
**Step-3:** To find Minimum Dominating Set from those vertices who are involving in feasible region and Domination number for the graph G.

- **Studying**

**Step-4:** Those vertices (in the form of $A(x_1, y_1), B(x_2, y_2)$ and etc ... ) involved in the minimum dominating set one of them is responsible for Optimal solution.
**Step-5:** Those vertices (in the form of $A(x_1, y_1), B(x_2, y_2)$ and etc ... ) are not involved in the minimum dominating set check whether they are create dominating set or not.

- **Testing**

**Step-6:** If dominating set satisfied the following conditions then it will give a optimized value of Z.
**Condition-1**
**Theorem-1:** Let S be a minimum dominating set and $|S| = \gamma(G)$ , domination number. If $x > y$ and x is not an integer, $y \in R$ then the optimum value of Z is generated by a vertex $A(x, y)$ which is a member of minimum dominating set and follows the condition $\left[\frac{y}{n}\right] \equiv [x] (\text{mod}\, \gamma(G))$ where n = total number of vertices of graph G means $|G| = n$.
**Condition-2**
**Lemma-1:** Let S be a minimum dominating set and $|S| = \gamma(G)$ , domination number. If $x < y$ and x is not an integer, $y \in R$ then the optimum value of Z is generated by a vertex $A(x, y)$ which is a member of minimum dominating set and follows the condition $\left[\frac{y}{n}\right] \equiv [x] (\text{mod}\, \gamma(G))$ where n = total number of vertices of graph G means $|G| = n$.
**Condition-3**
**Lemma-2:** Let S be a minimum dominating set and $|S| = \gamma(G)$ , domination number. If $x = y$ and x and y both are integers, then the optimum value of Z is generated by a vertex $A(x, y)$ which is a member of minimum dominating set and follows the condition $[\,y\,] \equiv [\,x\,] (\text{mod}\, \gamma(G))$ where n = total number of vertices of graph G means $|G| = n$.

**Condition-4**

**Lemma-3:** Let S be a minimum dominating set and $|S| = \gamma(G)$ , domination number. If $x \geq y$ and x and y both are integers, then the optimum value of Z is generated by a vertex A(x, y) which is a member of minimum dominating set and follows the condition $[\,y\,] \equiv \left[\frac{x}{n}\right] (\text{mod}\ \gamma(G))$ where n = total number of vertices of graph G means $|G| = n$.

**Condition-5**

**Lemma-4:** Let S be a minimum dominating set and $|S| = \gamma(G)$ , domination number. If $x \leq y$ and x and y both are integers, then the optimum value of Z is generated by a vertex A(x, y) which is a member of minimum dominating set and follows the condition $\left[\frac{y}{n}\right] \equiv [\,x\,](mod\ \gamma(G))$ where $n$ = total number of vertices of graph $G$ means $|G| = n$.

**Step-7:** As per above testing procedure a vertex satisfied one of the conditions 1 to 5 then we will obtained normal feasible solution of LPP using graphical method.

- Use

Generally by using graphical method we are able to discuss only basic feasible solution only while according to domination model technique we are capable for discussing about optimum basic feasible solution of LPP and minimize the limitations of Graphical method.

***3.2 Proof of the theorem and related example***

**Theorem-1:** Let $S$ be a minimum dominating set and $|S| = \gamma(G)$ , domination number. If $x > y$ and $x$ is not an integer, $y \in R$ then the optimum value of $Z$ is generated by a vertex $A(x,\ y)$ which is a member of minimum dominating set and follows the condition $\left[\frac{y}{n}\right] \equiv [x](mod\ \gamma(G))$ where $n$ = total number of vertices of graph $G$ means $|G| = n$.

**Proof:** Here given that $S$ be a minimum dominating set and $|S| = \gamma(G)$ .Now we assume that $x > y$ and $x$ is not an integer, $y \in R$.

**To Prove:** optimum value of $Z$ is generated by a vertex $A(x,\ y)$ which is a member of minimum dominating set and follows the condition $\left[\frac{y}{n}\right] \equiv [x](mod\ \gamma(G))$ where $n$ = total number of vertices of graph $G$ means $|G| = n$.

i.e. : Let $\left[\frac{y}{n}\right] = s$ , $[x] = t$ and $mod\ \gamma(G) = r$, where $s, t, r \in Z$. Therefore $s - t = rk$ wheere $k$ be any non negative integer.

Suppose $s - t \neq rk$ And given that $x > y$ follows $x - y > 0$

Here we are assume that without loss of generality $[x] - \left[\frac{y}{n}\right] = k > 0$

Therefore, $\frac{s-t}{r} \neq k$ means the ratio of two non negative integer is not an integer, that is contradict with our assumptions .

Therefore $s - t = rk$.

Means if $S$ be a minimum dominating set and $|S| = \gamma(G)$ , domination number. If $x > y$ and $x$ is not an integer, $y \in R$ then the optimum value of $Z$ is generated by a vertex $A(x,\ y)$ which is a member of minimum dominating set and follows the condition $\left[\frac{y}{n}\right] \equiv [x](mod\ \gamma(G))$ where $n$ = total number of vertices of graph $G$ means $|G| = n$.

According to theorem- 1 we can prove lemma 1 to 4, using same logic.

Vol. 8, Issue: 4, April: 2020
ISSN:(P) 2347-5404 ISSN:(O)2320 771X

**For example**

**Problem-1:**

Maximize $z = 3x + 4y$

Subject to

$4x + 2y \leq 80$

$2x + 5y \leq 180$

And $x, y \geq 0$

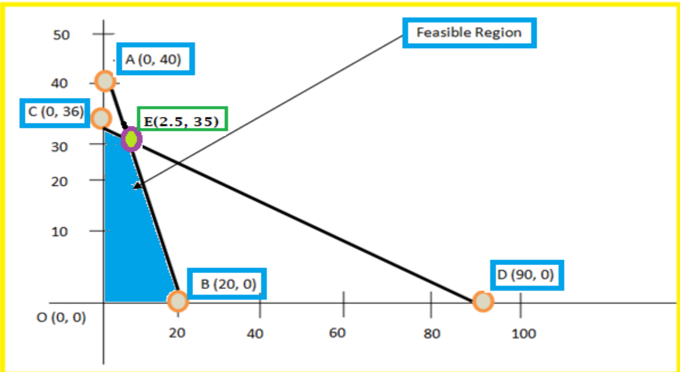
**Solution -1**

Step -1: Convert all constraints inequality to equality

| $Line - 2:\ 2x + 5y = 180$ | | |
|---|---|---|
| $x$ | 0 | 90 |
| $y$ | 36 | 0 |

| $Line - 1:\ 4x + 2y = 80$ | | |
|---|---|---|
| $x$ | 0 | 20 |
| $y$ | 40 | 0 |

Step -2: List the points and plot on the graph




The points are $A(0,40), B(20,0), C(0,36)\ and\ D(90,0), E(2.5,35)$.

**Step-3: Optimal solution of LPP**

| **Sr.No** | **Feasible Region** | **Optimize** $\boldsymbol{Max. z = 3x + 4y}$ |
|---|---|---|
| 1 | $O(0,0)$ | 0 |
| 2 | $B(20,0)$ | 60 |
| 3 | $\boldsymbol{E(2.5,35)}$ | **157.5** |
| 4 | $C(0,36)$ | 144 |

So the optimize value of $Z$ is 157.5.

Domination Model of Problem-1:

- **Building :**
- Feasible Region is OBEC
- OEBC are consider as vertices
- The minimum dominating set is $S = \{E\}$
- **Studying :**
- The set $S = \{E\}$ is responsible for optimum value.
- **Testing :**
- Here **Lemma -1** is applicable because in the vertex $E(x,y) = E(2.5,35)\ as\ 2.5 < 35$
- Therefore $E(2.5,35)$ is must satisfied the condition $[x] \equiv \left[\frac{y}{n}\right] (\text{mod}\ \gamma(G))$

- $\left[\frac{y}{n}\right] \equiv [x](\text{mod } \gamma(G)) \Rightarrow \left[\frac{35}{4}\right] \equiv [2.5](\text{mod } 1) \Rightarrow 8 \equiv 2(\text{mod } 1)$ it is true
- Therefore vertex E is responsible for optimum value and it is 157.5
- **Use :**
- From the domination model we are ensure about the optimize value for given LPP.

## 4. Conclusion

Using the domination model techniques we are able to minimize the following limitations of graphical method for solving LPP.

## 5. Limitations of Graphical Method in Linear Programming

- Linear programming is applicable only to problems where the constraints and objective function are linear i.e., where they can be expressed as equations which represent. In real life situations, when constraints or objective functions are not linear, this technique cannot be used.
- Factors such as uncertainty, weather conditions etc. are not taken into consideration.
- There may not be an integer as the solution, e.g., the number of men required may be a fraction and the nearest integer may not be the optimal solution. i.e., Linear programming technique may give practical valued answer which is not desirable.
- Only one single objective is dealt with while in real life situations, problems come with multi-objectives.
- Parameters are assumed to be constants but in reality they may not be so

## References

1. B. Andreas. Graphs with unique maximal clumpings. J. GraphTheory, 2:19 – 24, 1978.
2. C. Berge. Theory of Graphs and its Application. Methuen, London, 1962.
3. C. F. De Jaenisch. Trait des Applications de l' Analyse Mathematique au Jeu des Echecs. Petrograd. 1862.
4. D. Bauer, F. Harary, J. Niemien, and C.L. Suffel. Domination alteration sets in graphs. Discrete Math., 47:153 – 161, 1983.
5. E. J. Cockayne and S. T. Hedetniemi. Towards a theory of domination in graphs. Networks,7:247-261, 1977.

   E. J. Cockayne, P. J. P. Grobler, S. T. Hedetniemi, and A. A. McRae. What makes an irredundant set maximal? J. Combin. Math. Combin. Comput. To appear.
7. E. J. Cockayne, R. M. Dawes, and S. T. Hedetniemi. Total domination in graphs. Networks, 10:211 – 219, 1980.
8. G. Domke, S. T. Hedetniemi, R. C. Laskar, and G. Fricke, Relationships between integer and fractional
9. J. R. Carrington, F. Harary, and T. W. Haynes. Changing and unchanging the domination number of a graph. J. Combin.Comput., the 9:57 – 63, 1991.
10. M. Aigner. Some theorems on coverings. Studia Sci. Math.Hunger., 5:303 – 315, 1970.
11. parameters of graphs. Graph Theory, Combinatorics, and Applications, John Wiley & Sons, Inc. 2 (1991) 371 – 387.
12. R. B. Allan, R. C. Laskar, and S. T. Hedetniemi. A note on total domination. Discrete Math., 49:7 – 13, 1984.
13. R. C. Brigham, P. Z. Chinn and R. D. Dutton, A study of vertex domination critical graphs. Technical
14. Report, University of Central Florida (1984).