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Application of Connected Dominating Set in Ad-hoc Networking System

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Abstract
An Ad-hoc network is one that is spontaneously formed when device connect and communicate with each other. It is mostly wireless local area network (WLANs). The devices communicate with each other directly instead of relying on a base station or access points as in wireless LANs for data transfer co-ordination. Each device participates in routing activity, by determining the route using the routing algorithm and shortest distance between two nodes concept forwarding data to other devices via this route. According to a research paper "Dominating set algorithms for wireless sen sor networks survivability", by Tayler Pino, Salimur Choudhury and Fadi AL-Turjman"[1]. We are trying to apply the same phenomena for different purpose. In this paper we are trying to measure a temperature of big hall, with size 50 feet-50 feet using sensors with fixed life span (High definition sensor with life span 50 minute)[9], for this purpose we need 16 sensors, 4-controllers and 1-principal controller at a time (by Temperature measurement theory [9]) according to hall size. Now the problem is "How to save energy of sensors? or How to maintain regular time span of the sensor? And how to chose fixed position of those sensors in a hall? For that particular solution of the mentioned problem we are using an algorithm for fixing the position of sensors and by using dominating set concept and permutation techniques trying to save energy of sensors.
Keywords: Dominating set, Minimal dominating set, Minimum dominating set, Domination number, Induced Sub-graph of a Graph G, connected dominating set, Minimum connected dominating set, connected domination number and Wireless networks.

1. Introduction

Here first we define the necessary terms those are mentioned as keywords, also give an example of each if necessary.

Definition 1.1: Let G be a graph and $V(G)$ be the set of all vertices of graph G and S be the subset of $V(G)$.Then the set is said to be dominating set of the graph G if and only if for any $w \in V(G) - S$ i.e.we can find at least one vertex $v \in S$ such that w is adjacent to v. For example, consider the following graph G, according to the definition of dominating set; the set $S = \{v_1, v_4\}$ is a dominating set.[7][67-72]

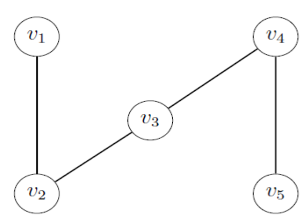

Figure 1: Dominating Set

Definition 1.2: Assume that G be a graph. A dominating set S in the graph G is called minimal dominating set in the graph G if and only if for any $v \in S$, $S-v$ is not a dominating Set in the graph G.[7] For example, consider the following graph G, in which according to the definition of minimal dominating set, the set $S = \{v_1, v_4\}$ is a minimal dominating set. [7][67-72]

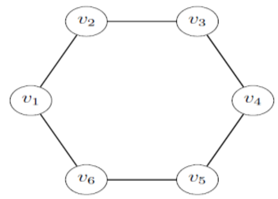

Figure 2: Minimal Dominating Set

Definition 1.3: Let G be any graph. A dominating set S in G with minimum cardinality is called a minimum dominating set of the graph G.A minimum dominating set in G is also called a $\gamma - set$ in the graph G. [8][51-55] For example, consider the following graph G, in which according to the definition of minimum dominating set, the set $S = \{v_1\}$ is a minimum dominating set.

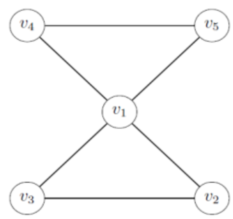

Figure 3: Minimum Dominating Set

Definition 1.4: Consider G be any graph and S being a minimum dominating set in G. Then $|S|$ means cardinality of set S, is called the domination number of the graph G and it is denoted by $\gamma(G)$. [7][67-72] For example, consider the following graph G, in which according to the definition of domination number, the set $S = \{v_1\}$ is a minimum dominating set and that cardinality is one therefore the domination number of graph G is equal to 1 means $\gamma(G) = 1$.

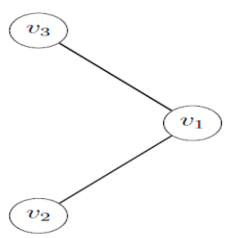

Figure 4: Domination Number

Remarks

(a) If S is a dominating set in graph G, then $\gamma(G) \leq |S|$. (b) Every minimum dominating set in G is a minimal dominating set in G.

(b) Every minimum dominating set in G is a minimal dominating set in G.

Definition 1.5: A sub-graph H of a graph G is said to an induced sub-graph, if it is obtained by deleting some vertices form graph G and the induced sub-graph of G is denoted by $G\ [X\ ]$. For example, Let us observe graph G from the following figure, now deleting the set of vertices $X = \{v_1, v_2\}$ then the remaining graph say

$H = G - \{X\}$ is called induced sub-graph of graph G. That is look like the following graph H which is shown in the following figures 5 and 6.

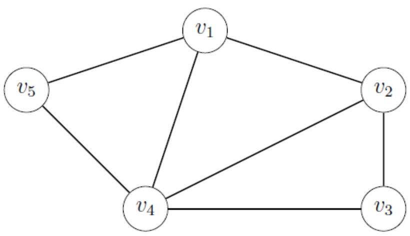

Figure 5: Graph G

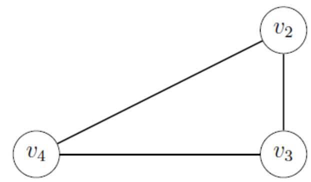

Figure 6: Induced Sub-graph of Graph G

Definition 1.6: A connected dominating set of a graph G is a dominating set whose induced sub-graph is also connected.

Definition 1.7: A minimum connected dominating set is a connected dominating set with smallest possible cardinality among all connected dominating set.

Definition 1.8: The connected domination number of graph G is the number of vertices in the minimum connected dominating set and it is denoted by $\gamma_c\ (G)$.

For example, consider a following graph G in the figure 7 in which the set $S = \{v_1, v_2\}$ is a dominating set. If we remove all the vertices except v_2 and v_3 then we are getting a induced sub-graph of graph G, and it is connected means there is an path between v_2 and v_3. Hence that is required connected dominating set (CDS).

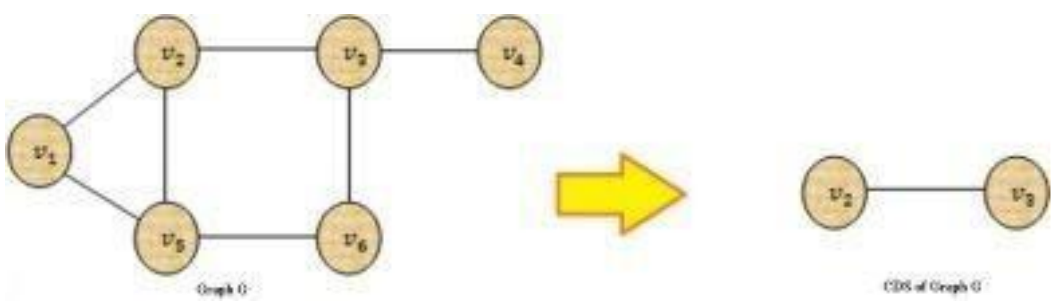

Figure 7: Graph G and its connected dominating set

Therefore $S = \{v_2, v_3\}$ is a CDS, shows in figure 6 and the connected domination number $\gamma_c\ (G\ ) = 2$.Here we are focusing on Ad-hoc networking system is a part of wireless network. So we can try to understand the concept of wireless network.

Definition 1.9: A network consists of two or more computers that are linked in order to share resources (such as printers and CDs), exchange files, or allow electronic communications. The computers on a network may be linked through cables, telephone lines, radio waves, satellites, or infrared light beams.[10]

Definition 1.10: The wireless network is a computer network that uses wireless data connections between network nodes.[10] For example: Cell phone network, WLANs, Wireless sensor network, Satellite network etc. There are two types of wireless networks showing in the following diagram with definitions and suitable examples.

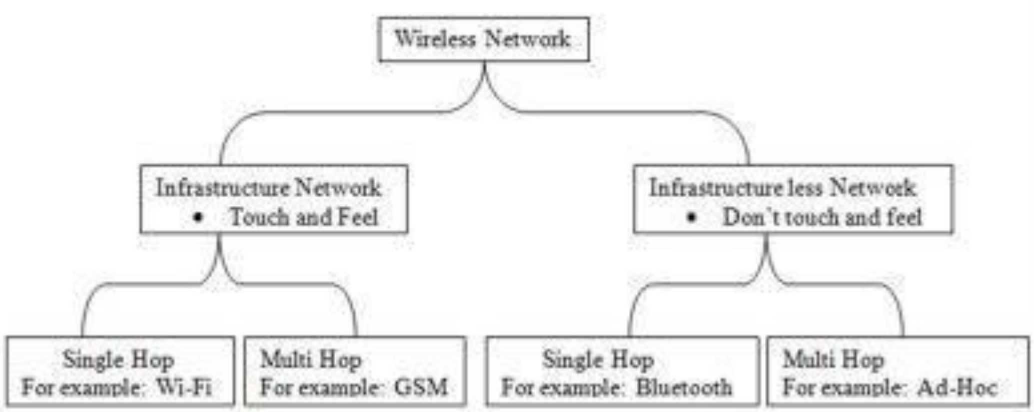

Figure 8: Types of Wireless Network

Definition 1.11: An Ad-hoc Network is infrastructure less multi hop network. In other words an ad-hoc network is a collection of wireless mobile hosts forming a temporary network without the aid of any stand-alone infrastructure or centralized administration.[10]

2. Main Results

Here we an try to prove the procedure for measuring the temperature of big hall at require time period without losing connectivity of sensors and saving energies of sensors becomes more easier by using ad-hoc network and connected dominating set concepts. For these purpose we defined some neces sary definitions, sensors arrangements and known results detailing below.

How to fixed position of the sensors on ruff?

As per mentioned in abstract now we are discussing about our first question as How to fixed position of the sensors on ruff?

Now let us choose the ruff of the big hall for arranging the sensors in which. Next we fixed the position of the sensors on top (ruff) of the big hall. By using permutation techniques, we are arranging the sensors on ruff in some particular sequence or order. According to a research paper under the title" Integrated Strategy for Generating Permutation", by Sharmila Karim, Zurni omar and Haslinda Ibrahim [2] we are able to do this in nice way. The whole procedure dividing into 6-steps as follows:

- Step-1: Let $S = \{s_1, ..., s_{(16=n)}\}$ as initial permutation and without loss of generality, the first element s_1, s_3 is fixed.
- Step-2: Identify the elements in the $(n - 2) = 14^{th}$ position of the initial permutation in step-1.Exchange this element until it reaches the $n = 16^{th}(last)$ position. Hereby four distinct starter sets are obtained in which we select $\{s_2, s_4, s_6, s_8\}$.
- Step-3: Identify the elements in the $(n - 6) = 10^{th}$ position of the initial permutation in step-2. Exchange this element until it reaches the n^{th} position. Hereby twelve distinct starter set are obtained in which we select $\{s_5, s_7\}$.
- Step-4:Identify the elements in the $(n - 8) = 8^{th}$ position of the initial permutation in step-3. Exchange this element until it reaches the n^{th} position. Hereby ten distinct starter set are obtained in which we select $\{s_9, s_{11}\}$.
- tep-5:Identify the elements in the $(n - 10) = 6^{th}$ position of the initial permutation in step-4. Exchange this element until it reaches the n^{th} position. Hereby eleven distinct starter set are obtained in which we select $\{s_{10}, s_{12}, s_{14}, s_{16}\}$.
- Step-6: Identify the elements in the $(n - 14) = 2^{th}$ position of the initial permutation in step-5. Exchange this element until it reaches the n^{th} position. Hereby seven distinct starter set are obtained in which we select $\{s_{13}, s_{15}\}$.

So as per the above algorithm we can fixed all 16 sensors, according to that order we can put a controller in which and fixed a direction to move on the data to controller. Now the final design is shown in following figure 9.

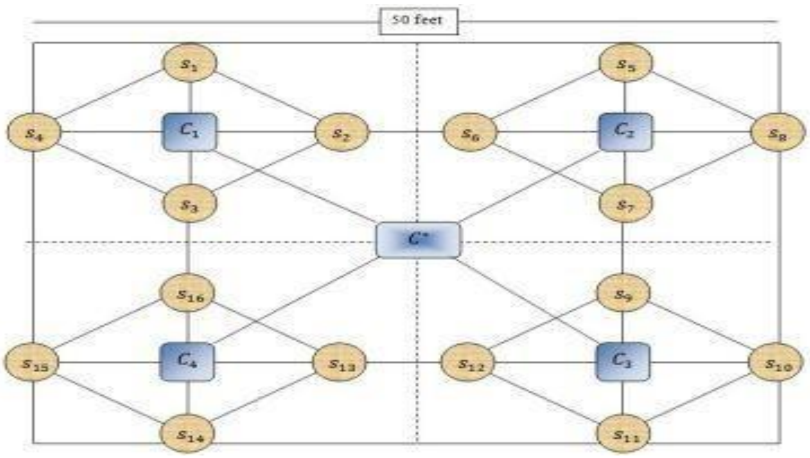

Figure 9: Design of sensors (Network) on the top of big hall

In above figure 8 the symbol $S_1, ..., S_{16}$ and $C_1, ..., C_4$ indicates sensors and controllers respectively while the role of C^* as a principal controller. Here we are using the LM 35 sensors, for measuring temperature. Its characteristic and original design as listed below.

(I) Characteristics of LM 35

- Linear+ 10-mV/°C Scale Factor
- 0.5 °C Ensured Accuracy(at 25 °C)
- Suitable for Remote Applications

- Low-Cost Due to Wafer-Level Trimming
- Less than 60-μA current drain
- A liberated directly in Celsius (Centigrade)

(II) LM 35 Design

For the purpose of measure the temperature of big hall we are using full range centigrade temperature sensor LM35 which is shown in the following figure 10.

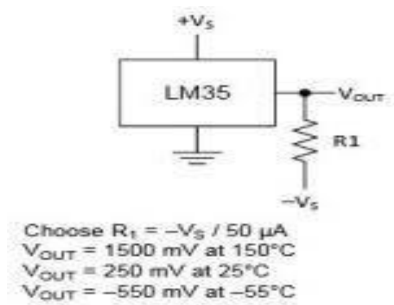

Figure 10: Full Range Temperature Sensor LM 35

List of some known results which are useful to fixed position of sensors

Lemma -1: The neighbor area of a vertex contains at most five independent vertices.

As per the figure 9, section 1, 2, 3 and 4 in which each vertex from section I connected with less than five vertices of another sections II, III and IV but not more than five independent vertices. i.e. $N(s_1) = \{s_4, s_2, C_1\}$ in which s_4 is independent from s_5, s_6, s_7, s_8, and C_2.

Lemma -2: The Unit arc triangle cannot contain two independent vertices.

Lemma -3: The neighbor area of two adjacent vertices contains at most eight independent vertices.

Lemma -4: Every tree T with at least three vertices has a non leaf vertex adjacent to at most one non leaf vertex.

How to get original data while some sensors lose their connectivity with controller?

Suppose at any case the sensors s_1, s_3 lost their connectivity with controller C_1 and vertex s_4 and the sensor s_8 lose their connectivity with vertices s_5, C_2 and $s_7$then situation is shown in following figure 11.

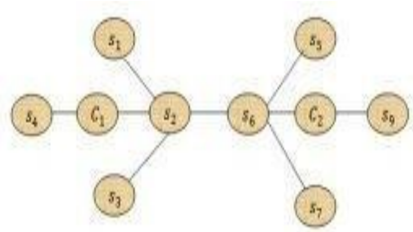

Figure 11: Case of lost connectivity of sensors

Therefore according to following lemma-4, we get a CDS and there is a one path between two vertices are always available so we will get a data through another path that is not break right now.

How to save energy of the sensors?

Now we can try to solve our next question is How to save energy of the sen sors?.As per above mentioned design we are create the clusters for measure ment of temperature. The main aim for creating the clusters is to maximize the time span and frequency of the sensors. So according to that, we are taking four partitions of the ruff of big hall mentioned in figure 10 now we connect the sensors of each partition with controller of the partition and that controller is also connecting with the center or principal controller.These all four clusters makes a connected dominating set with connected domination number one.It means that at any circumstances either power supply or con nectivity, a sensor and a controller can lose their connectivity then the data is always sure and safe with principal controller because there is more than one path between the sensors are always exists.If we want to get the data from the principal controller at after 10 minutes of time period then we set a switch in each sensors.So after sending the data into controller the respective sensor will be off otherwise on. This is the best way to serve energy of sensors and its life span also. Now all these matter can put into mathematical form as follows

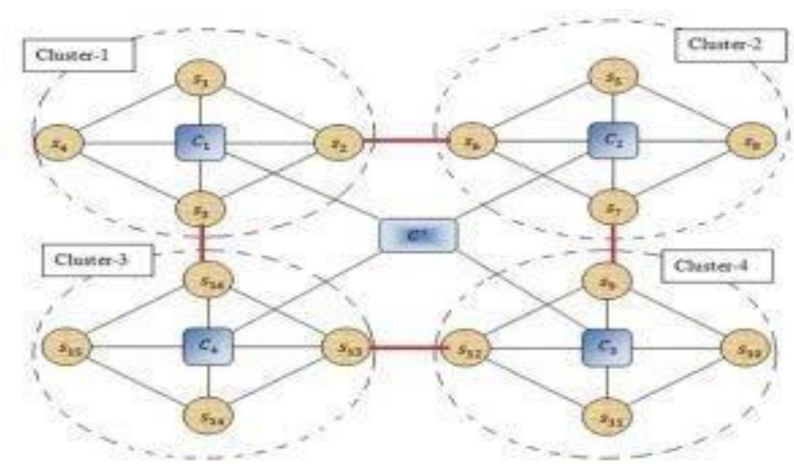

Figure 12: Clusters of sensors

Now we can try to obtain minimum connected dominating set from the above clusters (figure 10) shown in the following figure 13

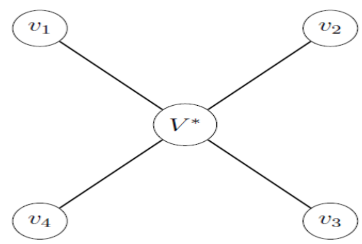

Figure 13: CDS obtained from Clusters

Above graph represent a wireless network with five nodes $C_1 = v_1$, $C_2 = v_2$, $C_3 = v_3$, $C_4 = v_4$ and $C^* = V^*$. Now the following definitions of the graph theory will help us to illustrate our problem.

Definition 2.1: An open neighborhood of vertex u is denoted by $N(u)$ defined as $N(u) = \{v/(u, v) \in E\}$ is the set of all nodes (vertices) that are adjacent with u.In above figure 11,the open neighborhood of vertex C^* is $N(C^*) = \{c_1, c_2, c_3, c_4\}$.[11]

Definition 2.2: The closed neighborhood of vertex u is denoted by $N[u]$ defined as $N[u] = \{v/(u, v) \in E\} \cup \{u\}$ is the set of all nodes (vertices) that are adjacent with u and u it self.In above figure 11,the open neighborhood of vertex C^* is $N[C^*] = \{c_1, c_2, c_3, c_4, C^*\}$.[11]

Definition 2.3: The maximum degree of vertex u is denoted by Δ defined as $\Delta\{u\}$ is the maximum number of edges incident with single node(vertex).In above figure 11,the $\Delta(C^*) = 4$ and figure 10 $\Delta(C_1) = \Delta(C_2) = \Delta(C_3) = \Delta(C_4) = 5$.[11]

Definition 2.4: Let $G = (V, E)$ be any graph then a set $S \subseteq V$ is said to be an independent set, if it contains non adjacent vertices in V. In figure 11, the set $S = \{C_1, C_2, C_3, C_4\}$ is an independent set.[11]

Definition 2.5: Let $G = (V, E)$ be any graph and a set $S \subseteq V$ is an independent set, is said to be maximal independent set, if by adding any vertex in S, can‟t breaks its independence property. In following figure 12, consider a graph G in which the set $S_1 = \{V_4, V_5, V_6, V_7\}$ is an independent set if we add the vertex V_2 in S, then the set S can't breaks its independence property therefore the set $S_2 = \{V_2, V_4, V_5, V_6, V_7\}$becomes maximal independent set.[11]

Theorem 2.1 : Every MIS is a dominating set[11] Remark
One approach to constructing a CDS is to find MIS and then add additional vertices (Nodes) as needed to connect the nodes in the MIS.

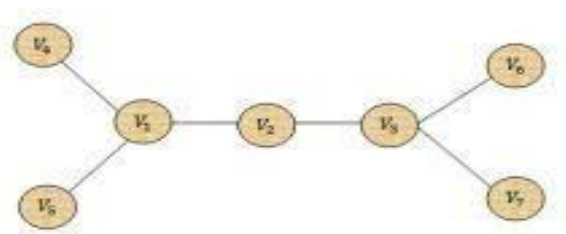

Figure 14: Graph G

Now in the above figure 10, the only neighbors are able to share and delivered the data to each other and all neighbors can make the MIS by adding one vertex(Node) from the another cluster. According to theorem 2 every MIS is dominating set, therefore that will create a dominating set, it means that $N(s_1) = \{s_2, s_4, C_1\}$, $N(s_2) = \{C_1, s_1, s_3, s_6\}$, $N(s_{16}) = \{s_3, s_{13}, s_{15}, C_4\}$ and $N(s_1) \cup \{s_3\}$ become MIS and that is dominating set. Those elements who are the members of MIS added one new node which is outside of the neighbor then we will get a induced sub-graph of graph G showing in the figure 14. That induced sub-graph is nothing but a CDS. Now the CDS (figure 13) of graph G figure 12, give us the nodes who are involve in the collection of data procedure so by help of CDS we are fully charged that sensors or controllers because only they are resources to collect the actual data. So the final conclusion is by help of CDS we can save energy of each and every sensors and controllers. So each controller having equal capacity of power storage that is$\Delta(C_1) = \Delta(C_2) = \Delta(C_3) = \Delta(C_4) = 5$.

3. Conclusion

An Ad-hoc networking system is very powerful tool for measuring tempera true, to set radio frequencies, connecting the more and more devices to each other and many more. The domination theory provided the mathematical background behind it.So we will try to merge two concepts and explain what is the beauty of CDS and MIS in Ad-hoc network using some known results and preliminary definitions.

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