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Open Packing Number of Triangular Snakes

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Abstract: A set $S \subseteq V(G)$ of vertices in a graph G is called a packing of G if the closed neighborhood of the vertices of S are pairwise disjoint in G. A subset S of $V(G)$ is called an open packing of G if the open neighborhood of the vertices of S are pairwise disjoint in G. We have investigated exact value of these parameters for triangular snakes.

Key Words: Neighborhood, packing, Smarandache k-packing, open packing.

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§1. Introduction

We begin with the finite, connected and undirected graph $G = (V(G), E(G))$ without multiple edges and loops. For a vertex $v \in V(G)$, the open neighborhood $N(v)$ of v is defined as $N(v) = \{u \in V(G)/uv \in E(G)\}$ and the closed neighborhood $N[v] = \{v\} \cup N(v)$. We denote the degree of a vertex $v \in V(G)$ in a graph G by $d_G(v)$. The minimum degree among the vertices of G is denoted by $\delta(G)$ and the maximum degree among the vertices of G is denoted by $\Delta(G)$. For any real number n, $\lfloor n \rfloor$ denotes the greatest integer not greater than that n and $\lceil n \rceil$ denotes the smallest integer not less than that n. For the various graph theoretic notations and terminology, we follows West [8] and Haynes et al. [3].

Definition 1.1 *The triangular snake T_n is obtained from the path P_n by replacing every edge of a path by a triangle C_3.*

Definition 1.2 *An alternate triangular snake AT_n is obtained from a path P_n with vertices $u_1, u_2, \cdots, u_n$ by joining u_i and u_{i+1} (alternately) to a new vertex v_i. That is every alternate edge of a path is replaced by C_3.*

Definition 1.3 *The double triangular snake $D(T_n)$ is obtained from a path P_n with vertices $v_1, v_2, \cdots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, \cdots, n-1$ and to a new vertex u_i for $i = 1, 2, \cdots, n-1$.*

Definition 1.4 *A double alternate triangular snake $D(AT_n)$ consists of two alternate triangular snakes which have a common path.*

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A packing of a graph G is a set of vertices whose closed neighborhoods are pairwise disjoint. Generally, a Smarandache k-packing of a graph G is a set of vertices whose closed neighborhoods intersect just in k vertices, and disjoint if $k = 0$. Equivalently, a packing of a graph G is a set of vertices whose elements are pairwise at distance at least 3 apart in G. The maximum cardinality of a packing set of G is called the packing number and it is denoted by $\rho(G)$. This concept was introduced by Biggs [1].

A subset S of $V(G)$ is an open packing of G if the open neighborhoods of the vertices of S are pairwise disjoint in G. The maximum cardinality of an open packing set is called the open packing number and is denoted by ρ^o. This concept was introduced by Henning and Slater [5]. A brief account of on open packing and its related concepts can be found in [2,4,6,7]. In the present paper, we obtain the packing and open packing number of various snakes.

§2. Main Results

Theorem 2.1 *For $n \geq 3$, $\rho(G) = \left\lceil \frac{n}{3} \right\rceil$, where G is triangular snake T_n and double triangular snake $D(T_n)$.*

Proof The triangular snake T_n is obtained from a path P_n with vertices $v_1, v_2, \cdots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, 3, \cdots, n-1$ while to construct double triangular snake $D(T_n)$ from a path P_n with vertices $v_1, v_2, \cdots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, 3, \cdots, n-1$ and to a new vertex u_i for $i = 1, 2, \cdots, n-1$.

If S is any packing set of G then it is obvious that v_1 must in S as $d_G(v_1) = 2 = \delta(G)$.

We construct a set S of vertices as follows:

$$S = \left\{ v_{3i+1} / 0 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1 \right\}$$

Then $|S| = \left\lceil \frac{n}{3} \right\rceil$. Moreover S is a packing set of G as $N[v] \cap N[u] \neq \phi$ for all $v, u \in S$. For any $w \in V(G) - S$, $N[v] \cap N[w] \neq \phi$ and $N[u] \cap N[w] \neq \phi$. Thus, S is a maximal packing set of G. Therefore any superset containing the vertices greater than that of $|S|$ can not be a packing set of G. Hence

$$\rho(G) = \left\lceil \frac{n}{3} \right\rceil. \quad \square$$

Theorem 2.2 *For $n \geq 3$, $\rho^o(G) = \left\lceil \frac{n}{3} \right\rceil$, where G is triangular snake T_n and double triangular snake $D(T_n)$.*

Proof The triangular snake T_n is obtained from a path P_n with vertices $v_1, v_2, \cdots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, 3, \cdots, n-1$ while to construct double triangular snake $D(T_n)$ from a path P_n with vertices $v_1, v_2, \cdots, v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, 3, \cdots, n-1$ and to a new vertex u_i for $i = 1, 2, \cdots, n-1$.

If S is any open packing set of G then it is obvious that v_1 must in S as $d_G(v_1) = 2 = \delta(G)$.

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$$\rho^o(G) = \left\lceil \frac{n}{3} \right\rceil. \qquad \square$$

Illustration 2.3 The graph T_7 and its packing number and open packing number are shown Figure 1 while the graph $D(T_7)$ and its packing number and open packing number are shown in Figure 2.

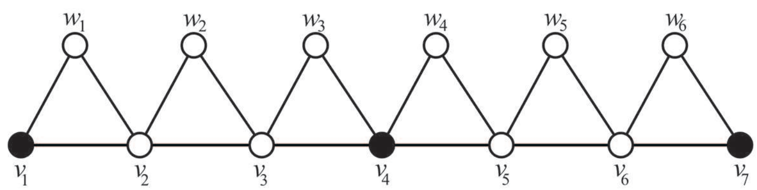

Figure 1 $\rho(T_7) = \rho^o(T_7) = 3$

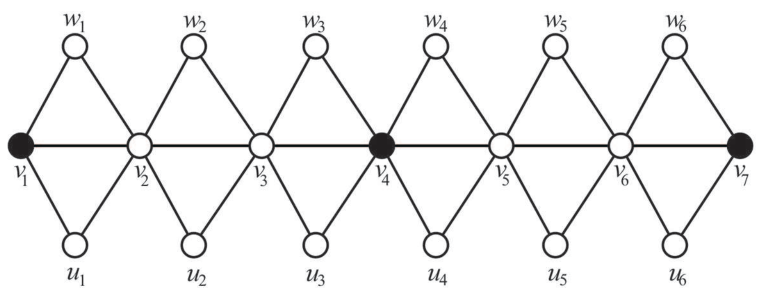

Figure 2 $\rho(D(T_7)) = \rho^o(D(T_7)) = 3$

Theorem 2.4 *For $n > 3$, $\rho(G) = \left\lceil \frac{n}{3} \right\rceil$, where G is alternate triangular snake AT_n and double alternate triangular snake $D(AT_n)$.*

Proof An alternate triangular snake AT_n is obtained from a path P_n with vertices $v_1, v_2, \cdots, v_n$ by joining v_i and v_{i+1} (alternately) to a new vertex w_i, $i = 1, 2, \cdots, n-1$ while to construct a double alternate triangular snake $D(AT_n)$ from a path P_n with vertices $v_1, v_2, \cdots, v_n$ by joining v_i and v_{i+1} (alternately) to a new vertex w_i, $i = 1, 2, \cdots, n-1$ and to a new vertex u_i for $i = 1, 2, \cdots, n-1$.

If S is any packing set of G then it is obvious that v_1 must in S as

$$d_G(v_1) = \delta(G) = \begin{cases} 1, & \text{if } n \text{ is odd,} \\ 2, & \text{if } n \text{ is even.} \end{cases}$$

We construct a set S of vertices as follows:

$$S = \left\{ v_{3i+1} / 0 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1 \right\}$$

Then $|S| = \left\lceil \frac{n}{3} \right\rceil$. Moreover S is a packing set of G as $N[v] \cap N[u] \neq \phi$ for all $v, u \in S$. For any $w \in V(G) - S$, $N[v] \cap N[w] \neq \phi$ and $N[u] \cap N[w] \neq \phi$. Thus, S is a maximal packing set of G. Therefore any superset containing the vertices greater than that of $|S|$ can not be a packing set of G. Hence

$$\rho(G) = \left\lceil \frac{n}{3} \right\rceil. \qquad \square$$

Illustration 2.5 The graph AT_7 and its packing number is shown Figure 3 while the graph $D(AT_7)$

and its packing number is shown in Figure 4.

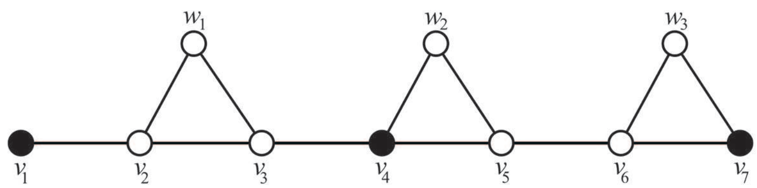

Figure 3 $\rho(AT_7) = 3$

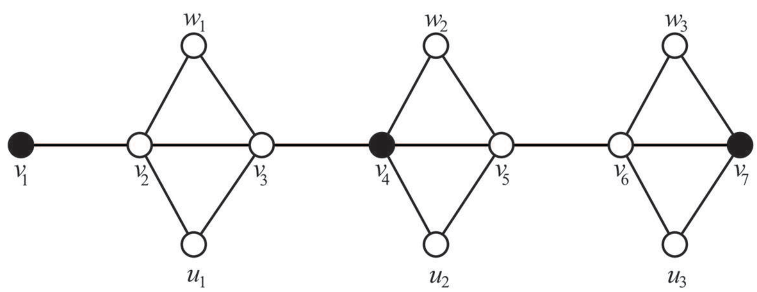

Figure 4 $\rho(D(AT_7)) = 3$

Theorem 2.6 *For $n > 3$, $\rho^o(G) = \left\lceil \frac{n}{2} \right\rceil$, where G is alternate triangular snake AT_n and double alternate triangular snake $D(AT_n)$.*

Proof An alternate triangular snake AT_n is obtained from a path P_n with vertices $v_1, v_2, \cdots, v_n$ by joining v_i and v_{i+1} (alternately) to a new vertex w_i, $i = 1, 2, \cdots, n-1$ while to construct a double alternate triangular snake $D(AT_n)$ from a path P_n with vertices $v_1, v_2, \cdots, v_n$ by joining v_i and v_{i+1} (alternately) to a new vertex w_i, $i = 1, 2, \cdots, n-1$ and to a new vertex u_i for $i = 1, 2, \cdots, n-1$.

If S is any open packing set of G then it is obvious that v_1 must in S as

$$d_G(v_1) = \delta(G) = \begin{cases} 1, & \text{if } n \text{ is odd}, \\ 2, & \text{if } n \text{ is even}. \end{cases}$$

We construct a set S of vertices as follows:

$$S = \begin{cases} \left\{ v_{4i+1}, v_{4i+2} / 0 \leq i \leq \left\lceil \frac{n}{5} \right\rceil \right\} & \text{for } n \text{ is odd} \\ \\ \left\{ v_{4i+2}, v_{4i+3} / 0 \leq i \leq \left\lfloor \frac{n}{5} \right\rfloor \right\} & \text{for } n \text{ is odd} \end{cases}$$

Then $|S| = \left\lceil \frac{n}{2} \right\rceil$. Moreover S is an open packing set of G as $N(v) \cap N(u) \neq \phi$ for all $v, u \in S$. For any $w \in V(G) - S$, $N(v) \cap N(w) \neq \phi$ and $N(u) \cap N(w) \neq \phi$. Thus, S is a maximal open packing set of G. Therefore any superset containing the vertices greater than that of $|S|$ can not be an open packing set of G. Hence

$$\rho^o(G) = \left\lceil \frac{n}{3} \right\rceil. \qquad \square$$

Illustration 2.7 The graph AT_7 and its open packing number is shown Figure 5 while the graph $D(AT_7)$ and its open packing number is shown in Figure 6.

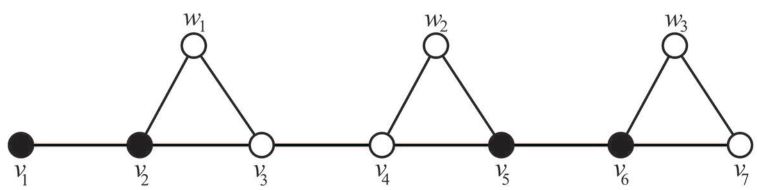

Figure 5 $\rho^o(AT_7) = 4$

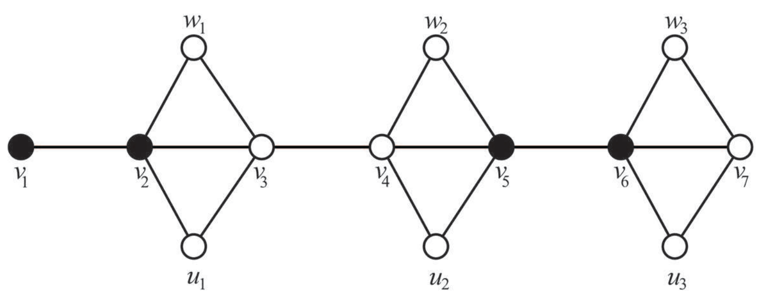

Figure 6 $\rho^o(D(AT_7)) = 4$

§3. Concluding Remarks

The concept of packing number relates three important graph parameters - neighborhood of a vertex, adjacency between two vertices and domination in graphs. We have investigated packing and open packing numbers of triangular snakes.

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References

[1] N.Biggs, Perfect codes in graphs, *J. Combin. Theory*, B(15)(1973), 289 - 296.

[2] I.S.Hamid, S.Saravanakumar, Packing parameters in graphs, *Discussiones Mathematicae*, 35(2015), 5 -16.

[3] T.W.Haynes, S.T.Hedetniemi, P. J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, 1998.

[4] M.A.Henning, D.Rautenbach, P.M.Schfer, Open packing, total domination, and the P_3-Radon number, *Discrete Mathematics*, 313(2013), 992 - 998.

[5] M.A.Henning, P.J.Slater, Open packing in graphs, *JCMCC*, 29(1999), 3 - 16.

[6] S.K.Vaidya, A.D.Parmar, Some new results on total equitable domination in graphs, *Journal of Computational Mathematica*, 1(2017), No. 1, 98-103.

[7] S.K.Vaidya, A.D.Parmar, Open packing number of some path related graphs, *Journal of Applied Science and Computations*, VI(V)(2019), 180-184.

[8] D. B. West, *Introduction to Graph Theory*, Prentice Hall, 2003.